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A Linear Programming Formulation for Multi-Region Planning of Electrified Powertrains Mix

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Executive Summary

Towards the goal of reduction of Greenhouse Gas (GHG) emissions, automotive manufacturers face several challenges when planning future vehicle offerings in different markets. The planned vehicle offerings must cope with uncertainties in the supply chains of critical materials, adhere to different regulatory requirements in different regions, all while appealing to customer preferences and maintaining low cost. Regulatory requirements, which are often based on tailpipe GHG emissions do not necessarily align with Lifecycle Analysis (LCA) of GHG emissions, which becomes yet another challenge towards attaining sustainability goals. Modeling all such considerations can be a complex task, but when considering a snapshot in time, such as the mix of vehicle powertrains in one future model-year, the decision-making process could be reduced into a linear programming (LP) problem that can be efficiently optimized. This paper presents the details of such formulation, along with practical examples for hybrid and electric vehicles in the US.

Keywords: Electric Vehicles; Plug-in Hybrid Vehicles; Hybrid Electric Vehicles; Supply and Value Chain; Modeling and Simulation.

1 Introduction

Vehicles with electrified powertrains [1], which include (non-plug-in) Hybrid Electric Vehicles (HEVs), Plug-in Hybrid Electric Vehicles (PHEVs) and Battery-only Electric Vehicles (BEVs) carry the promise of significant LCA GHG reductions compared to Conventional Internal Combustion Engine (CICE) vehicles. Electrified vehicles also include Hydrogen-powered Fuel Cell Electric Vehicles (FCEVs) [2] and Plug-in Fuel Cell Electric Vehicles (PFCEVs) [3]. However, fuel cells are relatively new technology that not only face challenges from vehicle design and manufacturing side, but also from readiness and availability of Hydrogen refueling infrastructure, as well as cost and supply chains of Hydrogen as a fuel [4-6]. As such, further discussions of electrified powertrains in this paper will be limited to HEVs, PHEVs and BEVs. HEVs may be considered as the “low end” or “entry level” electrified powertrain; having one or more electric motor(s), a “small capacity” traction battery, an internal combustion engine (ICE) and no means to charge the battery from the electric grid. Fuel consumption in the ICE of HEVs is the only source of energy, much like CICEs, and yet, compared to CICEs, HEVs have efficiency improvement features, including: i) ability to recapture vehicle kinetic energy while decelerating (also known as “regenerative braking”) by operating the motor(s) as generator(s) and storing the recaptured energy in the battery for later re-use, and ii) ability to take advantage of electric motors’ high efficiency across a broad range of torque and speed output requirements, which in turn allows the ICE to either operate close to its optimum conditions, or get briefly turned off, with the difference in traction power being compensated by the battery and electric motor(s). With those efficiency improvement features, it is estimated that HEVs are capable of achieving up to 25% – 35% reduction in tailpipe GHG emissions compared to equivalent CICEs [7, 8]. HEVs also present the least compromise from the perspective of tentative new adopters, who only need to “trust that the new technology will not fail prematurely”, since

HEVs are refueled and operated the same as CICEs, and do not require developing/managing a “new activity” such as charging of the vehicle. HEVs also require the least amount of battery materials since they utilize small capacity batteries compared to PHEVs and BEVs.

At the other end of the spectrum of electrified powertrains, BEVs may be considered the “high end” or “advanced level” among electrified powertrain; with no ICE at all, higher powered electric motor(s) and large-capacity traction battery that gets charged from the electric grid when the vehicle is parked at a charger. BEVs not only have the same energy efficiency features of HEVs (regenerative braking and high efficiency electric motors), but also have the capability for larger reduction in LCA GHG by relying on grid electricity as the energy source, which depending on the electricity generation mix in of the grid, is often of lower Carbon Intensity (CI) compared to Gasoline or other typical liquid fuels. However, in addition to requiring larger amounts of battery materials per manufactured vehicle, BEVs also face higher challenges in mass market adoption due to various reasons, including: higher initial cost (primarily due to larger batteries), as well as concerns/perceptions about driving range, charging time and availability of charging infrastructure [9-11].

As a plausible “middle ground”, PHEVs combine traits of both HEVs and BEVs. Though the components in the powertrain of a PHEV maybe be similar to that of HEV, key distinctions include the ability of PHEVs to charge their batteries via electricity from the grid (similar to BEVs), as well as PHEVs generally having larger battery capacities compared to HEVs. Under favorable conditions of low CI electricity, consistent charging and infrequent long trips, the LCA GHG of a PHEV could approach or even excel compared to that of an equivalent BEV [12, 13]. PHEVs also relieve range anxiety in long-distance travel since they can operate like HEV (consuming fuel in the ICE as the energy source) when the battery runs out. However, when conditions are unfavorable, such as little or no charging, the LCA GHG of a PHEV could become slightly worse than an equivalent HEV.

This paper considers an optimization model for planning the of future mix of powertrains among CICEs, HEVs, PHEVs and BEVs for minimization of both LCA GHG and cost, all while adhering to regulatory requirements, supply chain limitations (especially battery materials) and projected market demand. For simplicity, the formulation considered in this work only considers one future model-year of vehicles at a time but is able to simultaneously consider multiple vehicle sizes and in multiple geographic regions, which in turn that may have regulatory requirements, such as the US federal standard [14] and ZEV mandate for a subset of US states [15]. We show that optimization of powertrain mix planning in the presence of many such types of constraints can be expressed as a Linear Program (LP) [16], which allows optimal solutions to be efficiently estimated. This paper started with an overview of electrified powertrains leading to motivation for the work. The remainder of the manuscript is organized as follows: section 2 presents the details of the generalized mathematical model; section 3 presents a small-scale example study that considers only one region, one vehicle size category and three powertrain alternatives (CICE, PHEV and BEV). The small example allows for step-by-step visualization of various considerations in the LP optimization process. Section 4 then presents a larger scale example with two geographic regions, two vehicle size categories (mid-sized sedan and small SUV) and four powertrain alternatives (CICE, HEV, PHEV and BEV) for a total of sixteen decision variables, where the LP optimization algorithm known as “Dual Simplex” [17] is shown to successfully find optimal solutions (mix of powertrains) across the two regions and the two size categories, not only for the minimization of LCA GHG, but also a second objective such as cost reduction, as well as generating Pareto-optimal solutions that explore the trade-off between two objectives. Summary of findings and future work are briefly discussed in conclusion of the paper in section 5.

2 Mathematical Model

2.1 Overview

The formulation adopted in this paper conforms with the standard LP form [16], recapped in Eqns. (1-3). This work further adopts the notation where boldface italic symbols denote vector quantities, capitalized boldface non-italic symbols denote matrix quantities and non-boldface italic symbols denote scalar values. The optimization goal in linear programming is to maximize a reward function, denoted by the symbol f in Eqn. (1) by finding the optimal combination among a set of decision variables denoted by the vector $\boldsymbol{x} = \{x_i\}$, with $i \in \{1, \ldots, N\}$ being an index for the decision variables. A vector of reward coefficients $\boldsymbol{c} = \{c_i\}$ in Eqn. (1), with c_i defining the expected amount of reward corresponding to one unit of the corresponding decision variable x_i. Thus, the reward function f can be simply calculated as the dot product of the vectors $\boldsymbol{c}$ and $\boldsymbol{x}$. The matrix $\mathbf{A}$ and vector $\boldsymbol{b}$ in Eqn. (2), along with the condition in Eqn. (3) that all decision variables must me non-negative, define the “feasible space” for the decision variables. An index $k \in \{1, \ldots, K\}$ is utilized to denote the rows of the matrix $\mathbf{A}$ (or elements of the vector $\boldsymbol{b}$).

$$\text{Maximize} \quad f = \boldsymbol{c} \bullet \boldsymbol{x} \tag{1}$$

$$\text{Subject To} \quad \mathbf{A}\,\boldsymbol{x} \leq \boldsymbol{b} \tag{2}$$

$$\boldsymbol{x} \geq \boldsymbol{0} \tag{3}$$

The LP formulation in Eqns. (1-3) can be efficiently optimized via well-established algorithms, popular among-which is the Dual-Simplex algorithm [17] which is the one adopted in this work. However, the challenge that highlights the contribution of this work, is that of translating the various considerations in future planning of a mix of powertrains into a form that fits the standard LP. Towards that goal, we utilize a structured partitioning of the decision variables such that for a given total planned production of a number of vehicles M, there is a decision variable y_{srp} with three subscript indices $s \in \{1, \ldots, S\}$, $r \in \{1, \ldots, R\}$, $p \in \{1, \ldots, P\}$ respectively defining the vehicle size category, region for deployment or sales, and the powertrain type or specification. In some of the examples, we adopt the notation of p = 1 indicating BEVs, p = 2 indicating PHEVs, p = 3 indicating HEVs and p = 4 indicating CICEs, but this notation can easily be adopted to incorporate BEVs and PHEVs with different electric range offerings or additional powertrain types. It follows that there exists a direct correspondence between y_{srp} and the standard form of LP decision variables, which is expressed as:

$$y_{srp} = x_{(s-1)\times R\times P+(r-1)\times P+p} \tag{4}$$

While elements of the matrix $\mathbf{A} = [a_{ki}]$ are typically expressed via the row and column indices (k and i respectively), the column index is replaced by the three indices of the corresponding decision variable y_{srp}, thus the elements of the matrix **A** will be expressed as $[a_{ksrp}]$. For convenience, the row index (number of constraints in the matrix **A**) is further partitioned by constraint type into:

- Fleet consistency constraints $k \in K^F$
- Resource-bound constraints $k \in K^E$
- Pre-dictated fleet fraction constraints $k \in K^D$

Details for implementation of the different types of constraints are discussed in sub-sections 2.2 to 2.4. Sub-section 2.5 discusses formulations for the objective function (elements of the vector $\boldsymbol{c}$ in Eqn. 1), while sub-section 2.6 discusses the adopted approach for generating Pareto-optimal (trade-off) solutions among two different objectives.

2.2 Fleet Consistency Constraint(s)

The decision variables y_{srp} represent a fraction of the total number of vehicles (M) that are to be manufactured (for a certain future model-year). As such, the sum of the decision variables must add up to one, which is expressed as:

$$\sum_{s=1}^{S}\sum_{r=1}^{R}\sum_{p=1}^{P} y_{srp} = 1 \tag{5}$$

The equality constraint from Eqn. (5) could be expressed in standard LP form as two inequality constraints where the sum must be ≤ 1 and ≥ 1. It follows that the corresponding elements of the matrix **A** and vector $\boldsymbol{b}$ for the fleet consistency partition (two inequality constraints) are expressed as:

$$a_{k_1^F srp} = 1 \ \forall s, r, p \ \ \& \ \ b_{k_1^F} = 1 \tag{6}$$

$$a_{k_2^F srp} = -1 \ \forall s, r, p \ \ \& \ \ b_{k_2^F} = -1 \tag{7}$$

2.3 Resource-Bound Constraints

This category of constraints allows for modeling situations where each manufactured vehicle in the future model-year that's being planning would "consume" a certain amount of a limited "resource". The effect of such type of constraint is that any "valid solution" to be generated by LP, will have to adhere that the mix of powertrains do not require more than the allotted allowance of the resource. This category of constraints can be utilized to model things like total available materials for making vehicles or things like tailpipe-based regulation GHG, where the unit of resource "consumed" to produce a vehicle is its GHG rating, while the net available "resource" is the target compliance for fleet average GHG.

2.3.1 Available Materials Constraint

When certain material resources are in limited supply, such as when an OEM already has limited contracts with battery suppliers, one could incorporate such limitation in the powertrain mix planning via this type of constraint. If the total available amount of materials for producing M vehicles is Φ, then the average available amount of materials for one vehicle is $\phi = \Phi/M$. Let ϕ_{sp} be the amount of materials needed to manufacture one vehicle of size category s with powertrain type p. Then the limited resource material constraint can be expressed in linear form as:

$$\sum_{s=1}^{S}\sum_{r=1}^{R}\sum_{p=1}^{P} y_{srp}\phi_{sp} \le \phi \tag{8}$$

Considering j as a rolling index for constraints within the partition (resource-bound constraints $k \in K^E$ in this case), the corresponding elements of the matrix **A** and vector ***b*** in the standard form LP are:

$$a_{k_j^E srp} = \phi_{sp} \;\; \forall s, r, p \;\; \& \;\; b_{k_j^E} = \phi \tag{9}$$

2.3.2 Regulatory GHG Constraint

Certain GHG regulations (such as the US Federal [14]) are based on specific tailpipe lab-test GHG certification numbers rather than LCA GHG. Let ψ_{sp} be the GHG certification for a vehicle of size s and powertrain type p, with ψ being the fleet average GHG that the regulation allows for the target model-year. A constraint ensuring the powertrain mix complies with regulation GHG could be expressed as:

$$\sum_{s=1}^{S}\sum_{r=1}^{R}\sum_{p=1}^{P} y_{srp}\psi_{sp} \le \psi \tag{10}$$

However, in some GHG regulations (such the US Federal [14]), the GHG allowance, which is the right-hand side of Eqn. (10) depends on type of the vehicle ("Car" or "Light Truck") as well as its size denoted by its floor area. Let ψ_s be the fleet average GHG allowance for the special case of all the produced vehicles were to be of size s. Then, for a mix of different vehicle sizes, the right-hand side of Eqn. (10) could be expressed as:

$$\psi = \sum_{s=1}^{S}\sum_{r=1}^{R}\sum_{p=1}^{P} y_{srp}\psi_{s} \tag{11}$$

Substituting from Eqn. (11) into Eqn. (10) and rearranging:

$$\sum_{s=1}^{S}\sum_{r=1}^{R}\sum_{p=1}^{P} y_{srp}\left(\psi_{sp} - \psi_{s}\right) \le 0 \tag{12}$$

Eqn. (12) can then be expressed in standard LP form for the values of elements of the matrix **A** and vector ***b*** as:

$$a_{k_j^E srp} = \psi_{sp} - \psi_{s} \;\; \forall s, r, p \;\; \& \;\; b_{k_j^E} = 0 \tag{13}$$

2.3.3 Region-Specific Regulatory GHG Constraint

Though not demonstrated via numerical examples in this paper, a more generalized form of Eqns. (10-13) could be derived for the case when the powertrain planning is being conducted for multiple regions that could different GHG regulations in place. By simply utilizing the subscript $r \in R^E$, where R^E is a subset of the full set of region indices $\{1, \ldots, R\}$, ψ_{srp} being the GHG certification for a vehicle of size s and powertrain type p in region r with ψ_{sr} being the GHG allowance for size s in region r, the generalized version of Eqn. (12) could be expressed as:

$$\sum_{r\in R^E}\sum_{s=1}^{S}\sum_{p=1}^{P} y_{srp}\left(\psi_{srp} - \psi_{sr}\right) \le 0 \tag{14}$$

2.4 Pre-Dictated Fleet Fraction Constraints

Due to certain considerations such as demand forecast or strategic planning, the task of powertrain mix planning could involve imposing upper bounds, lower bounds or both upper and lower bounds on certain types of powertrains and/or vehicle sizes, for one or more geographic regions. Technology forcing type of regulations (such as the ZEV mandate [15]) could also be accommodated via constraints following this construction.

2.4.1 Generalized Less-Than Fleet Fraction Type Constraint

Let R^{D1} be a subset of the region indices $\{1, \ldots, R\}$, S^{D1} be a subset of the vehicle size indices $\{1, \ldots, S\}$ and P^{D1} be a subset of the powertrain type indices $\{1, \ldots, R\}$. Let R^{D2} be another subset of the region indices (could be the same as R^{D1}, or a different subset of indices), S^{D2} be another subset of the vehicle size indices (could be the same as S^{D1}, or different), P^{D2} be another subset of powertrain type indices (could be the same as P^{D1}, or different) and let α^{D1D2} be the target fraction not to be exceeded. The generalized form of the constraint could be expressed as:

$$\sum_{r\in R^{D1}} \sum_{s\in S^{D1}} \sum_{p\in P^{D1}} y_{srp} \le \alpha^{D1D2} \sum_{r\in R^{D2}} \sum_{s\in S^{D2}} \sum_{p\in P^{D2}} y_{srp} \tag{15}$$

Although this fully generalized form could be difficult to interpret, some of its special cases are more relatable and will be explored in sub-sections 2.4.3 to 2.4.5. Expressing Eqn. (15) in standard LP form for the values of elements of the matrix **A** and vector ***b*** can be done as:

$$a_{k_j^D srp} = \begin{cases} 1-\alpha^{D1D2} & \forall\{s,r,p\} \in \{S^{D1}, R^{D1}, P^{D1}\} \& \in \{S^{D2}, R^{D2}, P^{D2}\} \\ 1 & \forall\{s,r,p\} \in \{S^{D1}, R^{D1}, P^{D1}\} \& \notin \{S^{D2}, R^{D2}, P^{D2}\} \\ -\alpha^{D1D2} & \forall\{s,r,p\} \notin \{S^{D1}, R^{D1}, P^{D1}\} \& \in \{S^{D2}, R^{D2}, P^{D2}\} \\ 0 & \text{otherwise} \end{cases} \tag{16}$$

$$b_{k_j^D} = 0$$

2.4.2 Generalized Greater-Than Fleet Fraction Type Constraint

Following the same notation as in sub-section 2.4.1, but considering β^{D1D2} as the target fraction not to be less than, the generalized form of the constraint could be expressed as:

$$\sum_{r\in R^{D1}} \sum_{s\in S^{D1}} \sum_{p\in P^{D1}} y_{srp} \ge \beta^{D1D2} \sum_{r\in R^{D2}} \sum_{s\in S^{D2}} \sum_{p\in P^{D2}} y_{srp} \tag{17}$$

Noting that the standard LP form must be expressed as "less than", rearranging Eqn. (17) for the values of elements of the matrix **A** and vector ***b*** can be done as:

$$a_{k_j^D srp} = \begin{cases} \beta^{D1D2}-1 & \forall\{s,r,p\} \in \{S^{D1}, R^{D1}, P^{D1}\} \& \in \{S^{D2}, R^{D2}, P^{D2}\} \\ -1 & \forall\{s,r,p\} \in \{S^{D1}, R^{D1}, P^{D1}\} \& \notin \{S^{D2}, R^{D2}, P^{D2}\} \\ \beta^{D1D2} & \forall\{s,r,p\} \notin \{S^{D1}, R^{D1}, P^{D1}\} \& \in \{S^{D2}, R^{D2}, P^{D2}\} \\ 0 & \text{otherwise} \end{cases} \tag{18}$$

$$b_{k_j^D} = 0$$

2.4.3 Special Case: One Size Category Not Exceeding Fraction of Total

One example of a pre-dictated fleet fraction constraint could be the fraction of sedan vehicles, which generally have lower LCA GHG than larger sized vehicles (such as SUVs, minivans and Pickup Trucks) when comparing same powertrain type, i.e. sedan HEV will have lower LCA GHG than SUV HEV, sedan BEV will have lower LCA GHG than SUV BEV. Though this would make "larger fraction of sedan" be a desirable direction for an optimization that seeks minimum LCA GHG, a counter-balancing market demand limit (with α^{sedan} being the maximum fraction of the fleet) type of constraint could be formulated as a special case of Eqn. (15) as:

$$\sum_{r=1}^{R}\sum_{s=sedan}\sum_{p=1}^{P}y_{srp} \le \alpha^{sedan}\sum_{r=1}^{R}\sum_{s=1}^{S}\sum_{p=1}^{P}y_{srp} \tag{19}$$

2.4.4 Special Case: One Region No Less than Some Fraction of Total

When planning the allocation of vehicles across multiple regions, a situation that may arise could be that one or some regions are "less favorable" be it due to higher operating costs, infrastructure situation or stringency level of local regulations. To prevent the LP optimization from considering unrealistic solutions that may completely divest from one or more regions ($r \in R^{D1}$), one may employ a strategic decision of maintaining some market share as a special case of Eqn. (17), expressed as:

$$\sum_{r\in R^{D1}}\sum_{s=1}^{S}\sum_{p=1}^{P}y_{srp} \ge \beta^{D1}\sum_{r=1}^{R}\sum_{s=1}^{S}\sum_{p=1}^{P}y_{srp} \tag{20}$$

2.4.5 ZEV Mandate Type Constraints

The ZEV mandate [15], for California and other states in the US that choose to follow California's standard, dictates that new vehicle sales for model-year 2026 to 2035 will need to have a minimum fraction of (β^{ZEV}) of BEVs or PHEVs (or fuel cell, which we are not considering in this work, but can be added in the formulation), with the value β^{ZEV} increasing each year until reaching 100% by 2035. The ZEV mandate also dictates that no less than 80% of β^{ZEV} must be BEVs (or fuel cell). Both of those constraints can be expressed as special cases of Eqn. (17) as:

$$\sum_{r\in \text{ ZEV-state}}\sum_{s=1}^{S}\sum_{p\in\{BEV,\ PHEV\}}y_{srp} \ge \beta^{ZEV}\sum_{r\in \text{ ZEV-state}}\sum_{s=1}^{S}\sum_{p=1}^{P}y_{srp} \tag{21}$$

$$\sum_{r\in \text{ ZEV-state}}\sum_{s=1}^{S}\sum_{p=BEV}y_{srp} \ge 0.8\beta^{ZEV}\sum_{r\in \text{ ZEV-state}}\sum_{s=1}^{S}\sum_{p=1}^{P}y_{srp} \tag{22}$$

2.5 Objective Functions

2.5.1 Minimization of LCA GHG

Estimation of the LCA GHG of various vehicle sizes, powertrain types and the regions where they'll be driven (some regional aspects, such as the CI of the electric grid do affect LCA GHG) is performed as a "preprocessing" step prior to LP optimization. Let λ_{srp} be the estimated LCA GHG (in g-CO2/mile) for a vehicle of size *s* and powertrain type *p* driven in region *r*, we define a "minimization of LCA GHG" objective f^L for Eqn. (1) as the maximization of LCA GHG reduction compared to the worst performer, denoted by ($\lambda^{\max}$) and identified as:

$$\lambda^{\max} = \max_{s,r,p}(\lambda_{srp}) \tag{23}$$

Values of elements of the vector ***c*** for standard form LP in Eqn. (1) can then be calculated as:

$$c_{srp}^{L} = \lambda^{\max} - \lambda_{srp} \ \forall s, r, p \tag{24}$$

2.5.2 Cost Minimization

Similar to the formulation in sub-section 2.5.1, if there were a cost μ_{srp} associated with choosing to allocate one vehicle of size *s* and powertrain type *p* into region *r*, then a cost minimization objective f^O for Eqn. (1) could be formulated as maximization of the cost reduction compared to the most costly choice denoted by ($\mu^{\max}$) and identified as:

$$\mu^{\max} = \max_{s,r,p}(\mu_{srp}) \tag{25}$$

Values of elements of the vector ***c*** for standard form LP in Eqn. (1) can then be calculated as:

$$c_{srp}^{O} = \mu^{\max} - \mu_{srp} \ \forall s, r, p \tag{26}$$

2.6 Multi-Objective Optimization

Upon completion of calculation of the various elements of the matrix **A** and vector ***b*** (defining constraints of the LP optimization problem), one may utilize the coefficients vector $\boldsymbol{c}^L$ (from Eqn. 24) to seek a solution that minimizes LCA GHG, or the coefficients vector $\boldsymbol{c}^O$ (from Eqn. 26) to seek a solution that minimizes cost. In order to seek additional solutions that potentially offer good/balanced compromise between minimum LCA GHG and minimum cost, one may formulate a "weighted" objective as:

$$\boldsymbol{c}^W = w\,\boldsymbol{c}^L + (1-w)\;\boldsymbol{c}^O \tag{27}$$

where the scalar parameter (w) in Eqn. (27) is a weighing factor that can be gradually changed from a value close to 1.0 (where solution of the LP optimization converge to minimum LCA GHG) towards a value close to 0.0 (where solution of the LP optimization converge to the minimum cost), with other solutions generated through the sweep on the scalar parameter w being candidate compromise (Pareto-optimal) solutions. For generic multi-objective optimization problems, weighing the objectives has the known limitation of being unable to discover Parto-optimal solutions that are not on the convex portion of a Parto-frontier [18]. However, for LP problems, the feasible domain is always a convex set [16], and with the objectives being linear in terms of the decision variables, the Pareto-frontier cannot have non-convex topology. As such, the weighted objectives approach has the capability to generate all Pareto-optimal solutions.

3 Simple Example Study

LCA GHG estimates utilized in this example study, as well as the sixteen-decision variable scaled-up study in section 4, are based on CarGHG vehicle models [19, 20] and listed in Table 1. CarGHG [19] is a free open-source tool that brings together other open-source tools; primarily FASTSim [21] for fuel economy simulations in order to estimate fuel and electric energy amounts during the use phase of the vehicles, as well as GREET [22] for estimation of the equivalent GHG for manufacturing of traction batteries (scaled by kWh-battery) and size generic estimates for manufacturing everything else in the vehicle. Though CarGHG also includes cost models from the perspective of a vehicle buyer, it is difficult to identify reliable cost estimates from a vehicle planner's point of view. As such, the cost coefficients in Table 1 are to be considered place-holder values (assuming higher levels of electrification come with additional costs) and are only meant to allow demonstration of the multi-objective aspect of the LP optimization in this paper. Table 1 also shows estimates for US federal GHG ratings and model-year 2030 fleet average GHG allowance, which differs depending on whether the vehicle is considered a car or light truck.

The example in this section considers only three choices for the powertrain type (BEV, PHEV & CICE), only the small SUV size category and only one region designation (California & ZEV states). It follows that the LP standard form has only three decision variables (x_1, x_2, x_3) respectively corresponding to the fraction of BEVs, PHEVs and CICEs. The fleet consistency constraint from Eqn. (5) could be rearranged so that x_3 is calculated as a dependent variable ($x_3 = 1 - x_1 - x_2$, $x_1 + x_2 \leq 1$). Although typical application of LP involves having all constraints and one objective simultaneously solved by the optimization algorithm [17], with only two independent decision variables (x_1, x_2), the problem can be visualized step-by-step, as shown in Fig. 1.

Fig. 1.a shows the initial feasible domain with the fleet consistency constraint) and simple bounds on the independent decision variables. Fig. 1.b shows the feasible domain becoming smaller as a result of adding a brief set of market forecasting constraints that every powertrain type should retain at least 5% of the total, i.e. ($x_1 \geq 0.05$, $x_2 \geq 0.05$, $x_3 \geq 0.05$). Fig. 1.c shows the effect of adding US federal GHG regulation, which corresponds to a constraint ($85\ x_2 + 240\ x_3 \leq 140$), which in turn, after substituting for x_3 in terms of x_1, x_2 becomes ($240\ x_1 + 155\ x_2 \geq 100$), as shown in Fig. 1.c. For model year 2030, the ZEV mandate would require the sum of fractions of BEVs and PHEVs be at least 68%, and that 80% of the 68% (i.e. 54.4%) must be BEVs, as shown in Fig. 1.d and Fig. 1.e, respectively. Lastly, a constraint in Fig. 1.f on available materials for traction batteries (symbol ϕ in Eqn. 8) being 80 kWh per vehicle, with 110 kWh required to manufacture one BEV or 18 kWh required to manufacture one PHEV.

The remaining feasible domain (shaded area in Fig. 1.f) is shown at higher resolution in Fig. 1.g with its corner vertices labeled 'A' through 'E'. Fig. 1.g also shows the direction vectors of the objective seeking to maximize LCA GHG reduction (f^L), and the objective seeking to maximize cost reduction (f^O). One notable observation from this example is that the optimum point for the objective f^L is the vertex 'B' (rather than vertex 'A'). This observation exemplifies the notion that while BEVs may bring about larger GHG reductions than PHEVs for each retailed vehicle, an even higher priority is to displace as many CICEs as possible with either BEVs or PHEVs, per the available resources (battery materials in this case). Also observed from Fig. 1.g, is that vertex ‘D’ represents a solution with lowest cost (minimum amount of BEVs and PHEVs) yet is still feasible in terms of compliance with regulations for the target model-year.

When applying the approach discussed in section 2.6 for generation of Pareto-optimal solutions, starting with the objectives weighing scalar *w* at a value close to 1.0, the result of LP optimization will return the vertex ‘B’. As the weighing scalar *w* is gradually decreased, this corresponds to the direction vector of the weighed objective gradually “rotating” from being aligned with f^L towards becoming aligned with f^O, and at some intermediate value of *w*, the LP optimization will converge to the vertex ‘C’. And then with continued decreasing of the value of *w*, the LP optimization will eventually converge to the vertex ‘D’, thereby generating the Pareto-optimal solutions shown in Fig. 1.h. Furthermore, from the properties of LP, when the direction vector of an objective is orthogonal to a facet (edge) of the feasible domain, all solutions along the facet are equally optimal. In other words, when a sequence of vertices ‘B’, ‘C’, ‘D’ are Pareto optimal, then all intermediate solutions on the edges BC and CD are also Pareto-optimal

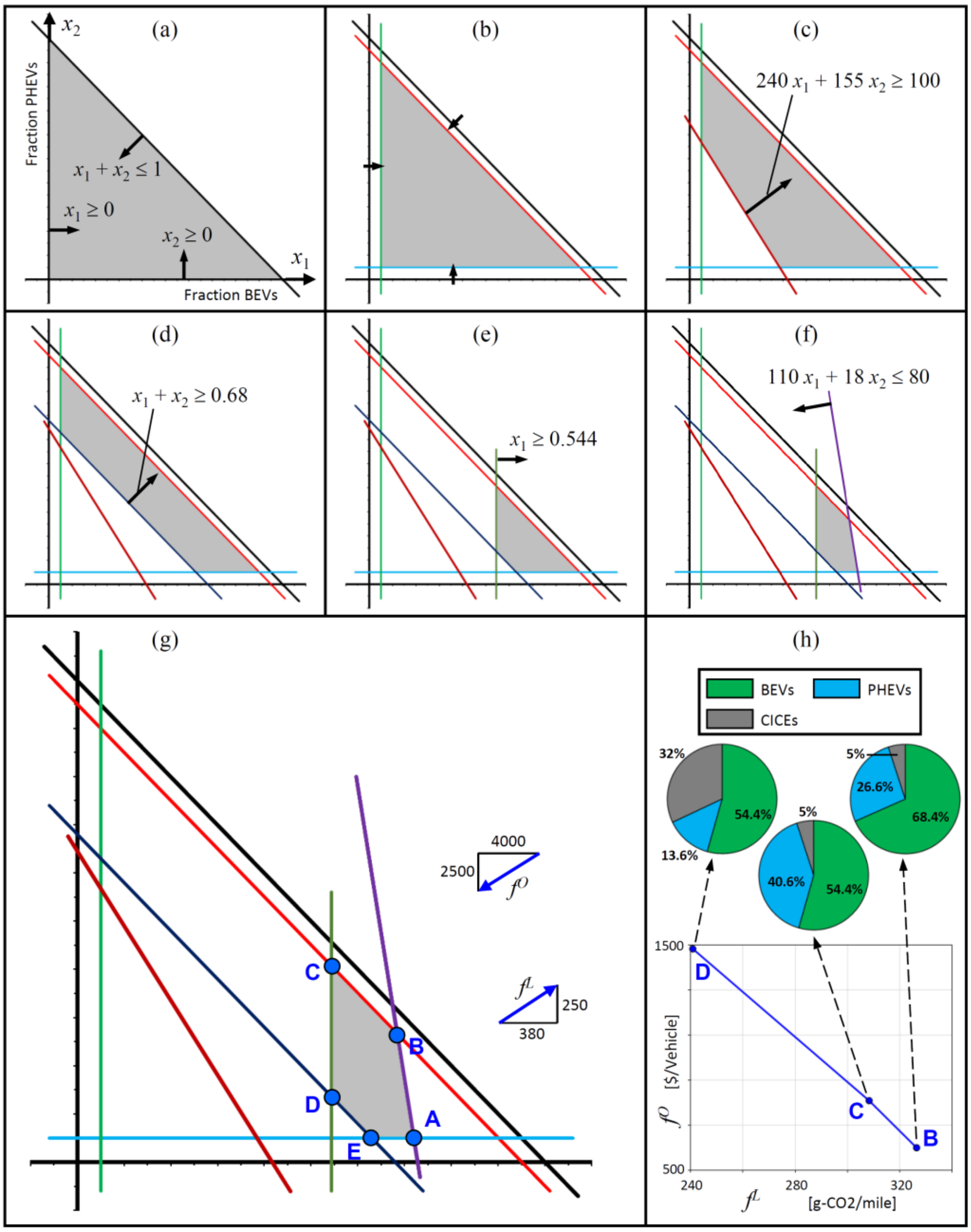

Figure 1: Step by step example LP optimization for two independent decision variables: (a) initial feasible domain, (b) to (f) reduction of feasible domain as constraints are added, (g) final feasible domain and objective function vectors, (h) Pareto-optimal solutions.

Table 1: Objective coefficients $\boldsymbol{c}^L$ and $\boldsymbol{c}^O$

Region r	Size s	Powertrain p	$c^L = \lambda^{max} - \lambda_{srp}$	$C^O = \mu^{max} - \mu_{srp}$	GHG rating	GHG allow.
1 = California & ZEV States	1 = Sedan	1 = BEV	400	0	0	100
		2 = PHEV	285	1000	45	100
		3= HEV	210	2000	150	100
		4 = CICE	90	3000	200	100
	2 = Small SUV	1 = BEV	380	0	0	140
		2 = PHEV	250	1500	85	140
		3= HEV	120	2500	175	140
		4 = CICE	0	4000	240	140
2 = Other US States	1 = Sedan	1 = BEV	350	0	0	100
		2 = PHEV	270	1000	45	100
		3= HEV	210	2000	150	100
		4 = CICE	90	3000	200	100
	2 = Small SUV	1 = BEV	320	0	0	140
		2 = PHEV	230	1500	85	140
		3= HEV	120	2500	175	140
		4 = CICE	0	4000	240	140

4 Scaled-Up Study

In this section, we expand the simple study from section 3 (for model-year 2023) to all the combinations of region, size category and powertrain type listed in Table 1. This includes California and ZEV states vs. other US states, models of mid-sized Sedan vs. Small SUV, and powertrain types {BEV, PHEV, HEV, CICE}, for a total of sixteen decision variables. In this case, the US federal GHG regulation constraint (via. Eqn, 14, with the GHG ratings and vehicle size GHG allowances from Table 1) is applied to the fleet average in all states. On the other hand, ZEV mandate constraints (sum of BEVs and PHEV ≥ 68%, BEVs ≥ 54.4%) are applied only within region 1. Market forecasting constraints were set that every powertrain type must maintain at least 5% of total vehicle sales in either size category segment, and to prevent region abandonment type solutions, additional constraints were imposed that at least 5% of California and ZEV state vehicles would be either HEV or CICE, while other US states must have at least 5% of their vehicles be BEVs and another 5% be PHEVs. Also, per recent data [23] indicating the share of total vehicle sales in California and ZEV states being approx. 40% of total vehicle sales in the US, bounds for the share of vehicles in region 1 were set between 35% and 45%. Furthermore, with the share of light-truck on a growing trend among light-duty vehicles sales in the US, reaching above 60% in 2023 [24], bounds for the share of small SUV were set between 50% and 70%. Lastly, the battery materials constraint was constructed per models for the vehicles in CarGHG [20] as sedans requiring 75 kWh, 12 kWh or 1.1 kWh to build one BEV, PHEV or HEV, respectively, while small SUVs requiring 110 kWh, 18 kWh or 1.2 kWh to respectively build one BEV, PHEV or HEV, with the average available battery materials per vehicle (symbol ϕ in Eqn. 8) being a study parameter for generating different scenarios. Results of LP optimization for the problem setup, including all the scenarios considered are shown in Fig. 2.

The considered scenarios include different levels of average available battery materials per vehicle (ϕ), designated by superscript Roman numbers {I, II, III, IV} for {80, 50, 30, 25} kWh values of ϕ, respectively. We also consider scenario symbols Θ and Ω, with Θ representing the problem with all ZEV mandate constraints, while the symbol Ω represents a scenario where the PHEVs cap (i.e. minimum BEVs constraint) is removed, thereby allowing more freedom in the decision-making about allocation of battery materials. For example, Θ^{II} is a scenario where $\phi = 50$ kWh and all the problem constraints included, while Ω^{IV} is scenario where $\phi = 25$ kWh and the constraint for PHEV cap in ZEV mandate removed. Fig. 2.a shows the Pareto plots (trade-off between f^L and f^O) for all the considered scenarios. However, with the cost coefficients in Table 1 being primarily placeholders, we focus discussion of results on the maximum LCA GHG reduction solution of each scenario, designated by an enlarged dot at the end of the curve plot in Fig. 2.a. Pie charts in Fig. 2.b show the details (fraction of vehicles by region, size and powertrain type) of the maximum LCA GHG reduction solution (enlarged dot at the end of the curve plot in Fig. 2.a) for the different scenarios. For higher level insights, Fig. 2.c, Fig. 2.d and Fig. 2.e respectively show summaries of the distribution of the vehicles by region, size category and powertrain type corresponding to the respective pie charts in Fig. 2.b.

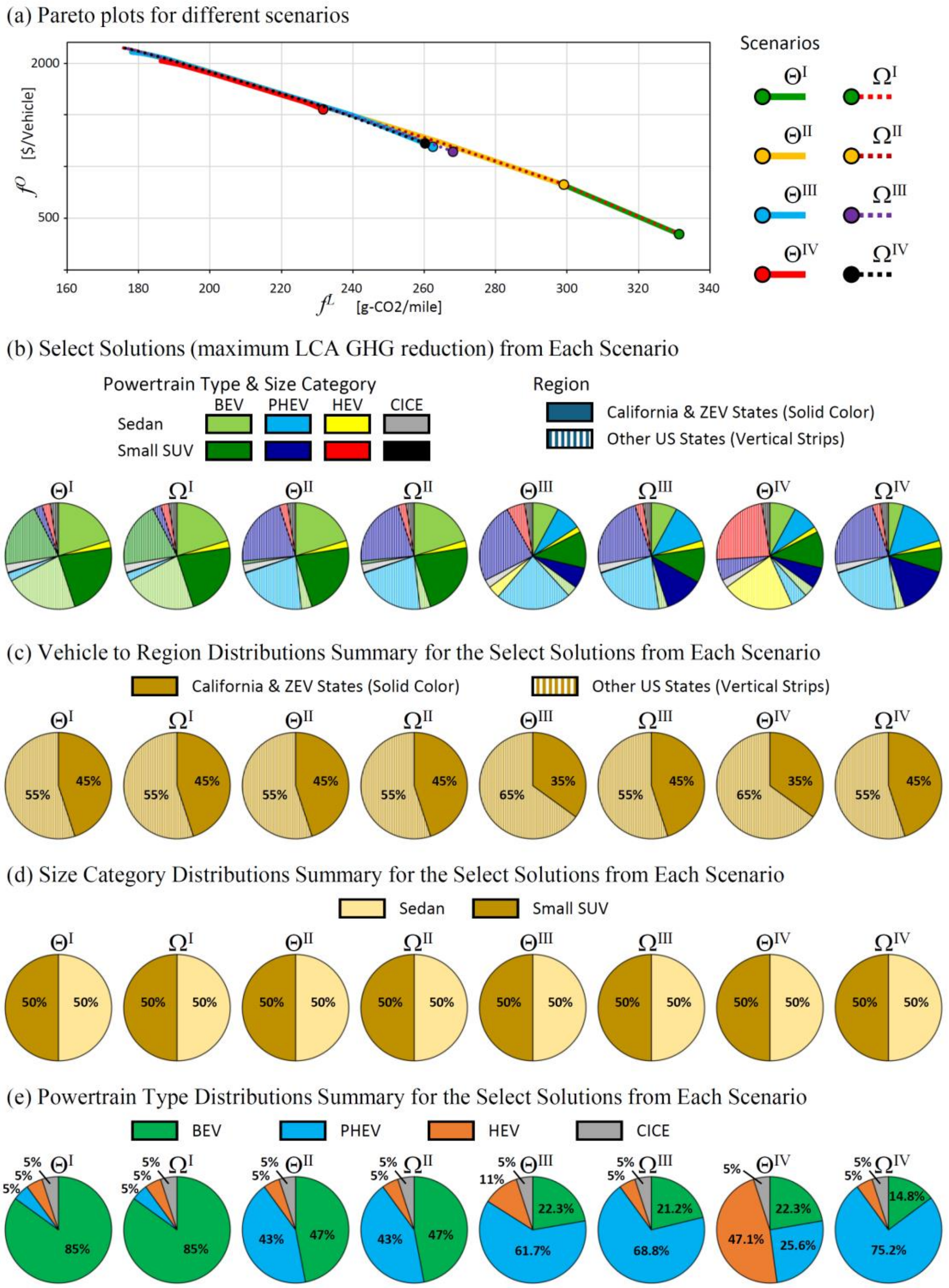

Figure 2: Results from scaled-up study with sixteen decision variables.

It is important to note that even this scaled up study could be considered significantly simplified when compared with a real powertrain planning task. However, the study serves to demonstrate the computational scalability of the proposed approach, as well as generating some observations that indicate that LP optimization is indeed making “sensible decisions” within the boundaries of the problem was setup. Some of the notable observations include:

- When the supply of battery materials is abundant (scenarios Θ^{I}, Ω^{I} in Fig. 2), the maximum LCA GHG reduction solution involves maximizing BEVs across all regions and size categories (Fig. 2.b). The optimum solutions in these scenarios also involve maximizing the share of vehicles in California and ZEV states (reaching the 45% maximum bound, as shown in Fig. 2.c for scenarios Θ^{I}, Ω^{I}), where the CI of the electric grid is lower, and thus BEVs have lower LCA GHG than BEVs in other US states.

- When the supply of battery materials becomes slightly less abundant (scenarios Θ^{II}, Ω^{II} in Fig. 2), the optimum solution prioritizes placement of BEVs in California and ZEV states (Fig. 2.b), while displacing as many CICEs as possible with the “next best” to BEVs (i.e. PHEVs) in other US states.
- When the supply of battery materials is not highly constrained, the optimum solution is not affected by whether the PHEV cap in the ZEV mandate is considered or not (comparing Θ^{I} vs Ω^{I} and Θ^{II} vs Ω^{II} in Fig. 2.b). However, in scenarios Θ^{III} and Θ^{IV}, meeting the ZEV mandate including the PHEV cap is only feasible by reducing the share of vehicles in California and ZEV states (reaching the 35% minimum bound, as shown in Fig. 2.c for scenarios Θ^{III}, Θ^{IV}) and an increasing share of HEVs in other US states, all of which result in less LCA GHG reduction compared to the respective scenarios of Ω^{III}, Ω^{IV} (where no PHEV cap is imposed) that allow shifting back the share of California and ZEV states to 45% (scenarios Ω^{III}, Ω^{IV} in Fig. 2.c) with maximum BEVs as allowed by available battery materials yet maximizing the displacement of CICEs via PHEVs.
- Though it might be challenging to achieve when considering the recent trend of consumer demand [24], all scenarios considered show it to be beneficial to reduce the fraction of larger sized vehicles (share of small SUVs at the minimum bound of 50% in Fig. 2.d).

5 Conclusion & Future Work

This paper presented a framework for formulating the decision-making process for future powertrain planning across multiple regions and vehicle sizes as a linear programming (LP) optimization problem. The LP formulation can incorporate generic limited resource type constraints and pre-dictated fleet fraction type constraints, which in turn, allow for practical modeling of many types of planning constraints, including regulatory GHG, ZEV mandate, materials supply and forecasted future demand. The framework is also capable of simultaneously optimizing two different objectives, thereby generating Pareto trade-off solutions. The proposed framework was demonstrated via a simple three-decision variable example, which was further reduced to two independent variables, thereby allowing for step-by-step visualization. A scaled-up study with sixteen decision variables, showcased LP optimization making sensible choices for different scenarios considered for the problem.

Future extension of this work may incorporate further scaled up studies with more vehicle size categories and/or further powertrain type options, such BEVs and PHEVs with different electric ranges. Other extensions of this work from a practical standpoint for future product planning could include adding consideration of uncertainties (within bounds and/or probability distributions) for the coefficient values of the objectives and/or constraints, as well as the consideration of multi-model-year product planning.

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Presenter Biography

Karim Hamza did his B.Sc. in Mechanical Design & Production (1998) and M.Sc. in Mechanical Engineering (2001) with a specialization in Robotics at Cairo University (Cairo, Egypt). He got his Ph.D. in Mechanical Engineering from the University of Michigan (2008, Ann Arbor, MI) with a dissertation focusing on design of vehicle structures for crashworthiness. Karim participated in several studies related to renewable energy and water desalination (post-doctoral research fellow at UM 2008-2012), did consulting work (2012-2014) for the future of mobility research division (FRD) at Toyota Research Institute of North America (TRINA), then joined TRINA full-time in 2015. His current research interests include modelling and analysis of electrified powertrains, as well as environmental impact and societal uptake of future mobility and transportation systems.